ASSESSMENT OF MAXIMUM HAIL SIZES AND THEIR VALUE AT A 100-YEAR RETURN PERIOD

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Abstract: *Hailstorms are relatively rare phenomena in Viet Nam, and understanding their characteristics is crucial for assessing the potential impact of hail events on various sectors. This study explores the maximum hailstone sizes observed in Viet Nam from 1961 to 2021 and uses the Gumbel distribution to estimate hailstone sizes at different return periods. Hail events in Viet Nam occur infrequently, with a common occurrence rate of less than once per year. However, some regions in the North, especially in the mountainous areas of the NorthWestern region, experience hail events approximately 1 - 2 times per year. The study records the largest hailstone sizes observed during the research period, with notable examples including 12 cm at the Bac Ha station and 10 cm at the Chiem Hoa station. These sizes exceed the calculated values for a 100-year return period, emphasizing the need for further research and preparedness. Despite the limited dataset of hailstone measurements, the Gumbel distribution is found to be a useful tool for estimating hailstone sizes at a 100-year return period. The theoretical estimates closely align with the observed values. At a 15-year return period, the most common maximum hailstone sizes range from approximately 1 - 3 cm, while at a 100-year return period, prevalent maximum hailstone sizes vary from 2.5 to 5.0 cm.*

Keywords: *Hailstorms, hailstone sizes, Viet Nam, Gumbel distribution, return periods.*

1. Introduction

Hail, a meteorological phenomenon characterized by the falling of frozen precipitation in the form of hailstones, is one of the natural disasters recognized under the Disaster Prevention and Control Law in Viet Nam. While relatively rare compared to other weather-related events, hail has the potential to cause significant damage to agriculture, property, and infrastructure.

Hailstorms often occur in conditions of atmospheric instability. This instability is characterized by the rapid vertical movement of air masses, which can lead to the development of strong updrafts within thunderstorms. These updrafts can carry raindrops high into the freezing upper levels of the atmosphere, where they freeze and form hailstones. Supercooled water is liquid water that remains in a liquid state below the freezing point. When supercooled water droplets come into contact with freezing nuclei (tiny ice particles or dust), they freeze instantly, contributing to hailstone formation. Viet Nam's tropical climate, with its warm surface temperatures, can create favourable conditions for supercooled water to exist in the upper atmosphere during thunderstorms. The strength of the updraft within a thunderstorm is a critical factor in determining hailstone size. Strong updrafts can support larger hailstone growth because they keep the hailstones suspended in the supercooled region for an extended period. Viet Nam's topography, including its mountainous terrain, can influence updraft strength, as air masses are lifted and accelerated over mountain ranges. Hail formation is influenced by

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temperature gradients in the atmosphere. A sharp decrease in temperature with height (known as lapse rate) promotes the development of hail.

In Viet Nam, while hail events are relatively infrequent compared to other weather-related phenomena, understanding the weather conditions that lead to hail is essential for forecasting, disaster preparedness, and risk mitigation. While hail events in Viet Nam are relatively rare, they can be triggered by specific meteorological factors such as atmospheric instability, supercooled water, strong updrafts, temperature gradients, moisture content, and convective activity. In June 2016, Hanoi, the capital city of Viet Nam, experienced a rare and severe hailstorm. The hailstones, some the size of golf balls, caused significant damage to vehicles, homes, and public infrastructure. The event served as a stark reminder of the unexpected nature of hailstorms in urban areas. Northern Viet Nam, including provinces such as Lao Cai and Son La, has been susceptible to hailstorms during the summer months. These hailstorms can devastate rice and vegetable crops, affecting local livelihoods and food security. Central regions of Viet Nam, including provinces like Quang Tri and Thua Thien Hue, have also experienced hailstorms [5].

Viet Nam has experienced severe hailstorms that have resulted in substantial damage to various aspects of life, including agriculture, infrastructure, and property. These extreme weather events, although relatively rare compared to other natural disasters in the country, have had far-reaching impacts on affected communities. In March 2019, the Dak Lak Province in the Central Highlands of Viet Nam witnessed a devastating hailstorm. Large hailstones, some reportedly as big as tennis balls, pelted the region, causing extensive damage to crops, particularly coffee plantations [5].

Hail size studies are a crucial component of meteorological research aimed at understanding the characteristics and behaviour of hailstones during hailstorms. These studies provide valuable insights into the size distribution, frequency, and variability of hailstones, which are essential for weather forecasting, disaster management, and infrastructure planning. Hailstones are categorized based on their size, typically measured in terms of diameter. The most common classifications include small (≤ 1 cm), medium (1 - 2 cm), large (2 - 5 cm), and giant (> 5 cm) hailstones [6]. Large hail [$25 mm (1 in.)] can produce significant damage to property and agriculture. However, little is known about the hazard posed by or incidence of the largest hail diameters [1].

In our previous study titled "Using observed hail to qualify extreme hail climate in Viet Nam. Part I: Data quality control and climatological characteristics of hail," published in the Journal of Climate Change Science No. 23, researchers explored the quality control of hail data and the climatological characteristics of hail in Viet Nam. This research laid the groundwork for a more comprehensive understanding of hail events in Viet Nam. By delving into the frequency and intensity of large hail events, we aim to enhance our capacity for hail forecasting and warning. This research is essential for improving the resilience of communities and industries in Viet Nam to the impacts of hailstorms and aligns with efforts to strengthen disaster prevention.

2. Data collection and methodology

2.1. Data

Long-term meteorological data spanning the period from 1961 to 2021 were obtained from 186 weather stations across Viet Nam. These stations represent various climatic regions within the country. The research presented here draws upon invaluable hail data collected as part of a study conducted by our previous study [2]. The data collected by Nguyen Dang Mau funded by FM Global [5] encompass a wide range of critical meteorological information related to hail events in Viet Nam. The dataset utilized in this research has undergone a meticulous standardization process for the extensive period from 1961 to 2021, covering data collected from 186 meteorological stations throughout Viet Nam (Figure 1). This comprehensive dataset represents a significant advancement in our ability to analyze and understand hail-related phenomena in Viet Nam.

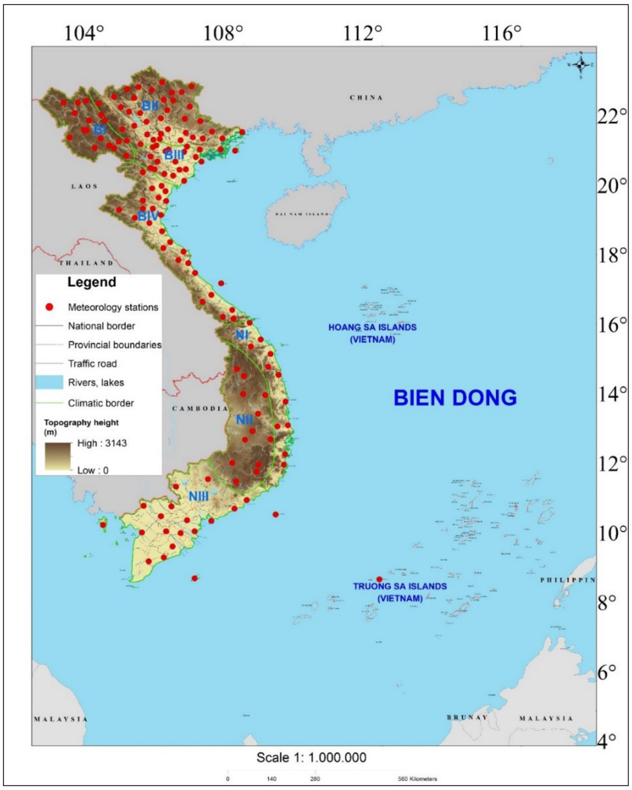

Figure 1. Distribution of meteorological stations

2.2. Methodology

a. Quality Control

Data quality control procedures were applied to ensure the accuracy and consistency of the meteorological data. This involved identifying and addressing errors, outliers, and missing values. This data quality control was implemented in our previous study "Using observed hail to quality extreme hail climate in Viet Nam. Part i: Data quality control and climatological characteristics of hail".

b. Statistical Analysis

Frequency Analysis: The frequency of hail events was analyzed, and the common occurrence rate of hailstorms was calculated for each station.

Gumbel Distribution: The Gumbel distribution, a widely used statistical method for extreme value analysis, was employed to estimate the maximum hailstone sizes at different return periods (e.g., 15-year and 100-year return periods) for each climatic region in Viet Nam.

The GEV distribution (Jenkinson 1955) is a continuous probability distribution that combines the Gumbel, Frechet, and Weibull families, also known as type I, II, and III extreme value distributions. The relationship is usually presented in the following form:

$$F(x) = exp\left\{-\left[1 + k\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/k}\right\} \qquad (1)$$

Where, in this application, represents the probability of occurrence of a given hail's size, and where *k, σ*, and *μ* are known as the shape, scale, and location parameters, respectively. For *k = 0*, Eq. (1) reduces to the Gumbel (EV1) distribution, whereas for positive and negative *k*, the distributions are, respectively, Frechet (EV2) and Weibull (EV3).

Where there is insufficient information about the extreme tail of a dataset, a popular first-order solution is to set *k = 0* and consider the simplest Gumbel (type I) distribution (Hosking et al. 1985). Thus, Eq. (1) in the type 1 case simplifies to:

$$F(x) = exp\{exp[(x - \mu)/\sigma]\} \qquad (2)$$

The location parameter μ summarizes the location or shift of the body of extremes (in this case the mean annual maximum hail's size), while the scale parameter σ describes its statistical dispersion (interannual variability of the annual maximum hail's size).

The first step is to fit a Probability Distribution Function (PDF) or Cumulative Distribution Function (CDF) to each group comprised of the data values for a specific duration. It is possible to relate the annual maximum hail diameter with the corresponding return period from the cumulative distribution function. Once a cumulative frequency is known, the maximum hail diameter is determined using the chosen theoretical distribution function (Gumbel distributions).

Dithering observational data:

We slightly modify the observed values in the data record by a small random number (drawn from a uniform distribution). This didn't change the overall statistical properties of the dataset. It helps to remove the saw-tooth pattern and the dithered data is more smooth.

X: input data

*a = -(0.247*x+0.0279);*
*b = 0.247*x+0.0279;*
a = max(a,-0.25);
b = min(b,0.25);
*r = a + (b-a)*rand(length(x),1);*

Exceedance probability of observation:

Calculate the exceedance probability associated with a specific observation

T = (N+1)/i

Where,

N = Number of annual maxima observations (35 in this case);

i = Rank of specific observation with *i = 1* being the largest to *i = N* being the smallest Column (C);

Parameter estimation:

To estimate the parameters of Gumbel distribution using the maximum likelihood estimation (MLE) method, we need to simultaneously solve the following two equations:

$$\beta = \bar{x} - \frac{\sum_{i=1}^{n} x_i \exp(-\frac{x_i}{\beta})}{\sum_{i=1}^{n} \exp(-\frac{xi}{\beta})}$$

$$\mu = -\beta \ln\left[\frac{1}{n}\sum_{i=1}^{n} \exp(-x_i/\beta)\right]$$

Deal with Zero value:

We adjust the effective return periods by including the zeros in the return period/ exceedance probability calculation. The return values for given return periods (T) can be calculated from the Gumbel quantile function (Gramosa et al. 2019):

*q = location - scale*log(-log(1-p^))*

Where,

p^=(p-w)/(1-w), p=1-1/T

weights *w = M/N*

M is the number of years with zeros

N is the total number of years

Goodness of Fit Test:

For chi-square goodness-of-fit computation, the data are divided into k bins and the test statistic is defined as

$$x^2 = \sum_{i=1}^{k} (O_i - E_i)^2/E_i$$

Where O_i is the observed frequency for bin *i* and E_i is the expected frequency for bin *i*. The expected frequency is calculated by

$E_i = N(F(Y_u) - F(Y_l))$

Where *F* is the cumulative distribution function for the distribution being tested, Y_u is the upper limit for class *i*, Y_l is the lower limit for class *i*, and *N* is the sample size.

Table 1. Chi-Square Distribution [8]

Chi-Square (x^2) Distribution

Degrees of Freedom	Area to the Right of Critical Value							
	0.99	**0.975**	**0.95**	**0.90**	**0.10**	**0.05**	**0.025**	**0.01**
1	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086
6	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475
8	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725
12	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217
13	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688
14	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141
15	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578
16	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000
17	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409
18	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805
19	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191
20	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566
21	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932
22	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289
23	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638
24	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980
25	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314
26	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642
27	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963
28	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278
29	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588
30	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892

3. Results

3.1. Climatolofy hail event

Frequency of hailstorms:

The analysis of hail frequency in Viet Nam, as indicated by the observational data presented in Figure 2 (left), reveals that hail occurrences are indeed rare events in the country. The data collected from 186 meteorological stations in Viet Nam indicate that hail has been observed at 125 of these stations. These stations are distributed across various regions of Viet Nam as follows:

- 22 stations in the NorthWestern region
- 45 stations in the Northeastern region
- 12 stations in the Red River Delta region
- 24 stations in the North Central region
- 6 stations in the South Central region
- 10 stations in the Central Highlands
- 6 stations in the Southern region

The observational data further demonstrate that the frequency of hail occurrences at most stations is less than once per year. The highest frequency of hail events in Viet Nam is observed in the NorthWestern region, where the majority of stations report hail events occurring more than twice per year on average. In contrast, the lowest frequency of hail events is found in the Southern region, with several stations having never observed hail during the recorded period.

These findings underscore the rarity of hailstorms in Viet Nam and the significant regional variations in hail frequency. The NorthWestern region stands out as the most hail-prone area in the country, with a relatively high occurrence of hail events, while the Southern region experiences the least frequent hail occurrences. Understanding these regional disparities is vital for disaster preparedness and risk assessment, as it highlights the need for tailored strategies to mitigate the impact of hailstorms in different parts of Viet Nam. Additionally, the data highlight the importance of continued monitoring and research to comprehensively assess the changing patterns of hail occurrences in Viet Nam and their potential relationship to broader climate trends.

Maximum historical hail size:

The analysis of maximum hailstone sizes recorded at meteorological stations in Viet Nam during the period from 1961 to 2021, as depicted in Figure 2 (right), provides valuable insights into the range of hailstone sizes observed across the country. The majority of meteorological stations have reported maximum hailstone sizes in the range of 1 - 3 cm, indicating that small to moderate -sized hailstones are the most common.

However, there have been instances of larger hailstone sizes, with some stations in the Northern mountainous region of North Viet Nam recording hailstones measuring up to 9 - 10 cm in diameter. Notable examples include the observation of 10 cm hailstones at the Bac Ha station on March 27, 2013, and at the Tuyen Hoa station on April 3, 2016.

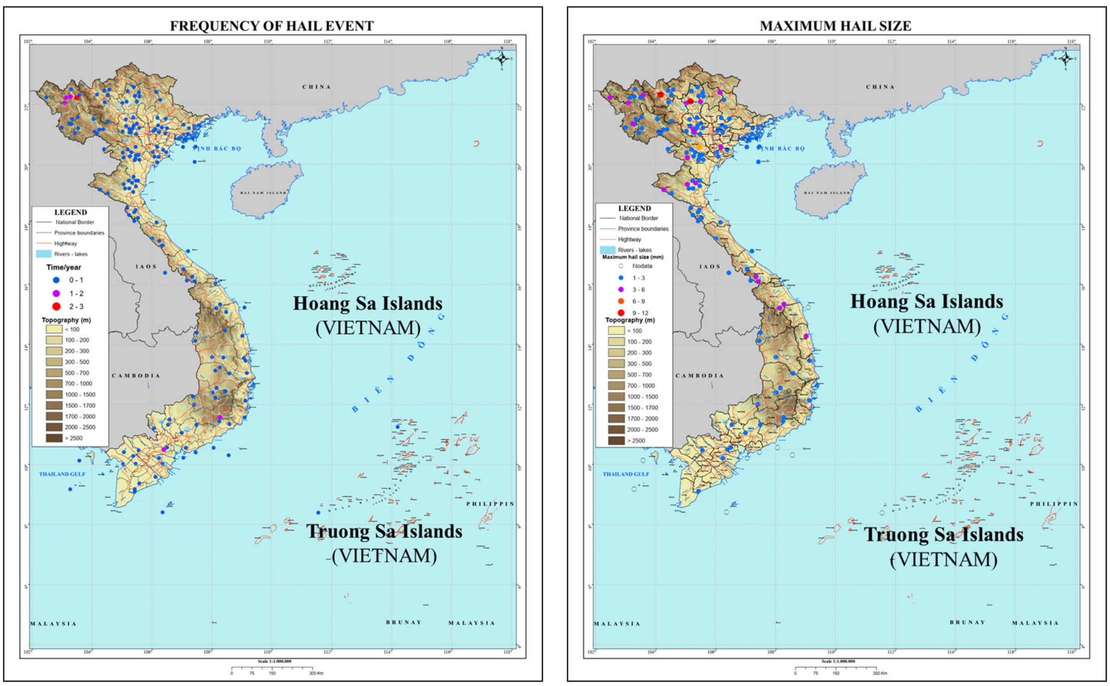

Figure 2. Mean frequency of hailstorm (event/year) (left) and maximum historical hail's size in cm (right)

In the Northern regions of North Viet Nam, hailstone sizes commonly range from 3 - 5 cm, with several stations reporting this size range. For instance, in the North Central region, hailstones measuring 3 - 5 cm were recorded at the Nhu Xuan station on March 18, 2017. In the Nghe An province, hailstones of 3 - 5 cm were observed on April 21 - 22, 2016, in Quy Chau. Additionally, in the Tuong Duong district, hailstones of the same size were reported on April 9, 2016. Moving to the South Central region, maximum hailstone sizes of up to 5 cm were recorded at the Tam Ky station on March 5, 2016, and at the Tra My station on April 23, 2016. In the Central Highlands region, the largest recorded hailstone size was 3 cm, observed at the Da Lat station. Finally, in the Southern region of Viet Nam, maximum hailstone sizes of 3 cm were recorded in Ho Chi Minh City in 2021.

3.2. Model fitting

The results of the model fitting for hailstone sizes using the Gumbel distribution, as presented in Figure 3 (for selected representative stations in various regions; all results presented in the study of Nguyen Dang Mau et al, 2022), offer valuable insights into the ability of the Gumbel distribution to capture the variability in hailstone sizes at selected meteorological stations across different climatic regions in Viet Nam. Despite the rarity of hail events and the considerable variation in hailstone sizes during different hailstorms, the Gumbel distribution demonstrates its effectiveness in characterizing hailstone sizes at stations with historical observations.

The Gumbel distribution, a statistical tool commonly employed to analyze extreme events, proves to be a suitable model for hailstone size distribution in various climate regions of Viet Nam. This finding is significant because it suggests that the Gumbel distribution can provide a useful framework for understanding and quantifying the occurrence of extreme hailstone sizes, even in regions where hail events are infrequent.

The capability of the Gumbel distribution to capture the variation in hailstone sizes is a valuable asset for meteorologists, climate scientists, and disaster management authorities. It allows for the estimation of probabilities associated with different hailstone size thresholds, which is crucial for risk assessment, disaster preparedness, and the development of mitigation strategies. Additionally, the use of the Gumbel distribution aids in establishing a more comprehensive understanding of hailstorm characteristics and their potential implications for climate studies and extreme weather event management.

Specifically, the calculated results for each climatic region in Viet Nam are as follows:

In the Northwest region, the distribution of the maximum hail diameter can be well captured by the Gumbel function. The Gumbel curves are not sensitive to the presence of outliers, for example in Bac Yen and Muong Lay for the stations Bac Yen, Mai Chau, Muong Lay, Muong Te, Phu Yen, and Tuan Giao. The Gumbel curves show a slight deviation from the observations of Mai Chau and Muong Lay, the Gumbel fitting curves seem to overestimate the observed values with a return period of 5 - 50 years while they underestimate the observed values with a return period greater than 50 years in these stations. In contrast, in Tuan Giao, the curve underestimates the observed values with a return period of 10 - 20 years while it overestimates the observed values with a return period greater than 30 years.

In the Northeast region, the distribution of the maximum hail diameter in many stations cannot be well captured by the Gumbel function because of outliers and the unusual shapes of the distribution of data points. In general, the curves follow the major observational data points nicely; however, they seem to underestimate the observed values from the return period greater than 50 years at some stations. The reason for such a large deviation between the theory lines and observation comes from the significant increase in hail size from the return period greater than 50 years. These stations are: Bac Can, Bac Giang, Bac Me, Bao Lac, Cao Bang, Cho Ra, Ha Giang, Huu

Lung, Lao Cai, Ngan Son, Mu Cang Chai, Phu Ho, Tuyen Quang, Viet Tri. In contrast, there is only one station (Chiem Hoa) where the curve overestimates the observation.

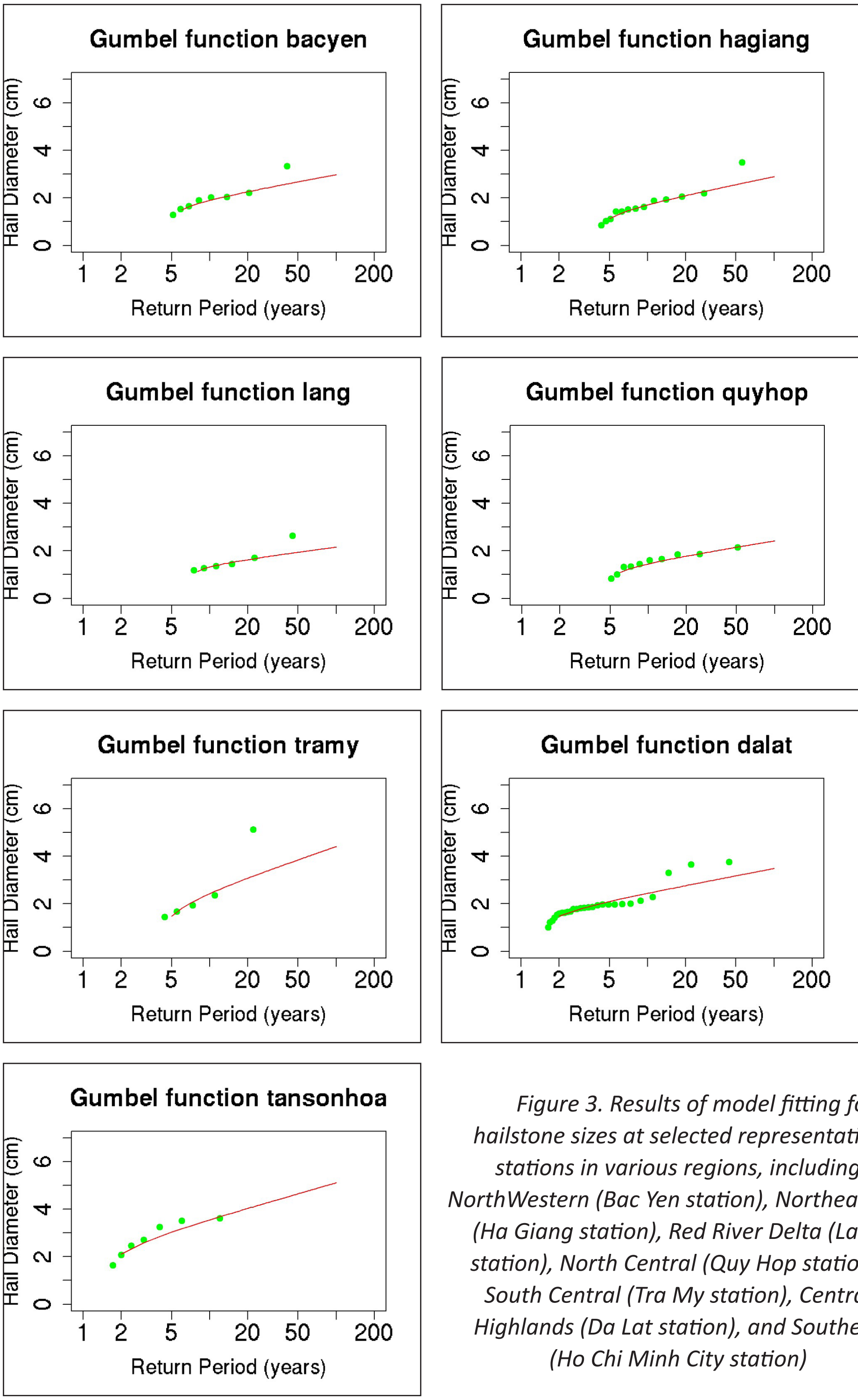

Figure 3. Results of model fitting for hailstone sizes at selected representative stations in various regions, including NorthWestern (Bac Yen station), Northeastern (Ha Giang station), Red River Delta (Lang station), North Central (Quy Hop station), South Central (Tra My station), Central Highlands (Da Lat station), and Southern (Ho Chi Minh City station)

In the Red River Delta, the Gumbel function nearly captures the observation at Lang station. In Ha Dong station, there is a sharp change in hail's size from the return period of 7 - 10 years making it difficult to be presented by the Gumbel curve. In Son Tay station, the Gumbel function follows closely the data points from the return period of 7 to 30 years but it underestimates the data points with the return period greater than 50 years.

In the North-Central region, there are two stations (Quy Hop and Tay Hieu) where the Gumbel curves closely follow the data points. In other stations, the Gumbel curves can well capture the observation in the return period from 5 - 50 years but it underestimated the data points with the return period greater than 50 years. In Tuong Duong stations, the curve underestimates the observation with a return period greater than 5 years.

In the South Central region, there is only one Tra My station having greater than 4 events. It seems that the Gumbel Curve closely follows the data points with the return period from 5 to 10 years. There is a large deviation between the data point with a return period of 20 years and the theory line.

In this Central Highlands region, it seems that the Gumbel function can capture the observed values with a return period of 3 - 20 years but it underestimates the values with a return period greater than 20 years in Kon Tum and Play Cu. The Gumbel curve underestimates the data points overestimating the data point with a return period from 5 to 10 years but underestimates the data point with a return period greater than 10 years. In Buon Me Thuot station, there is a large deviation between the theory line and observations.

In the Southern region, there hail event was observed at Tan Son Hoa with 4 events during the study duration. In this station, it seems to be that the observed values can be well captured by the Gumbel function. although there is a slight difference between the theory line and observation in the return period of around 5 years.

3.3. The maximum hail's size at a 100-year return period

The results presented in Figure 4 provide an analysis of the calculated maximum hailstone sizes at a 100-year return period using the Gumbel distribution for various climatic regions in Viet Nam. Specifically, the findings for each region are as follows:

• NorthWestern Region: The most common hailstone sizes in the NorthWestern region typically range from 2.3 to 3.6 cm, with the largest observed hailstones reaching 4.3 cm at the Muong Lai and Tuan Giao stations.

• Northeastern Region: In the Northeastern region, the most prevalent hailstone sizes range from 2.0 to 5.0 cm. The largest recorded hailstone size was 8.3 cm at the Chiem Hoa station, followed by 7.5 cm at the Bac Ha station. Notably, the largest observed hailstone size (12.0 cm) significantly exceeds the calculated hailstone size at the 100-year return period.

• Red River Delta Region: Hailstone sizes in the Red River Delta region commonly range from 2.2 to 8.1 cm, with the largest hailstones recorded at the Ha Dong station.

• North Central Region: In the North Central region, hailstone sizes typically range from 2.5 to 5.0 cm, with the largest hailstones observed at the Tuong Duong station.

• South Central Region: The largest hailstone size recorded in the South Central region was 4.9 cm, detected only at the Tra My station.

• Central Highlands (Tay Nguyen): Hailstone sizes in the Central Highlands region mostly fall within the range of 2.5 to 3.5 cm.

• Southern Region: The largest hailstone size in the Southern region was 5.1 cm, recorded at the Ho Chi Minh City station.

These results offer insights into the potential maximum hailstone sizes at a 100-year return period for different climatic regions in Viet Nam. It is important to note that while the Gumbel distribution provides valuable estimations, there may still be cases where observed hailstone sizes exceed the calculated values, as indicated in the Northeastern region. Understanding the distribution of hailstone sizes at extreme return periods is essential for assessing the highest possible impact of hailstorms and enhancing disaster preparedness and resilience strategies in each region.

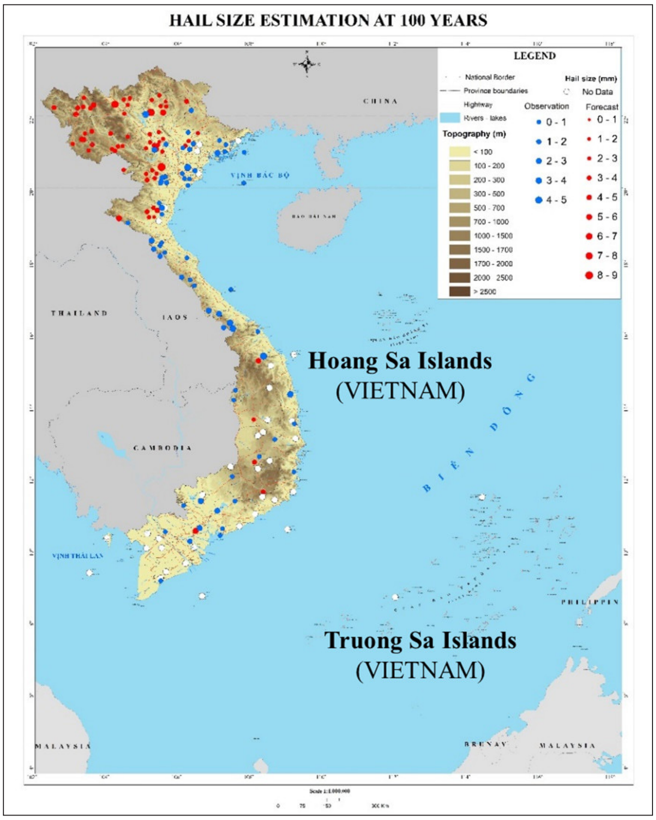

Figure 4. Calculated results for the largest hailstone sizes at a 15-year return period at 186 meteorological stations in Viet Nam

4. Concluding remarks and discussion

The results of this study on maximum hailstone sizes in Viet Nam spanning the years 1961 to 2021 provide valuable insights into the characteristics of hail events in the country. The key findings of the research are as follows:

Hailstorms in Viet Nam are infrequent, with a common occurrence rate of less than once per year. Some stations in the Northern region, particularly in the mountainous areas of Northern Viet Nam, experience hail events approximately 1 - 2 times per year.

The largest observed hailstone sizes in Viet Nam during the study period were 12 cm at the Bac Ha station and 10 cm at the Chiem Hoa station. These extreme hailstone sizes significantly exceeded the calculated values for a 100-year return period.

Despite the limited sample size of hailstone measurements, the Gumbel distribution proved to be a useful tool for estimating hailstone sizes at different return periods. The theoretical estimates closely approximated the observed values.

At a 100-year return period, the prevalent maximum hailstone sizes ranged from 2.5 to 5.0 cm. Some stations recorded even larger hailstones, with values of 7.5 cm (Bac Ha station), 8.1 cm (Ha Dong station), and 8.3 cm (Chiem Hoa station).

This study sheds light on the rarity of hail events in Viet Nam and their associated hailstone sizes. While the Gumbel distribution provides valuable estimates for hailstone sizes at different return periods, it is important to note that observed hailstone sizes can exceed these calculated values, as demonstrated by the extreme events recorded at certain stations. This research contributes to our understanding of hailstorm characteristics in Viet Nam and can aid in improving risk assessment and disaster preparedness strategies for hail-related events in the country.

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